Differentiation exercise – show differential equation

1. If $y = x \sin 2x$, prove that $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + \frac{2y}{x} + 4xy = 0$

2. Given that $y = e^x - e^{-x}$, show that $\left(\frac{dy}{dx}\right)^2 - y^2 - 4 = 0$

3. Given that $v = \sqrt{\sin u}$, show that $4v^3\frac{d^2v}{du^2} + v^4 + 1 = 0$

4. Given $y = e^{-x}\cos x$, show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0.$

5. Given that $y = \frac{\sin kx}{1+\cos kx}$, where k is a positive integer, show that $\sin kx \frac{d^2y}{dx^2} = k^2y^2.$

6. Given $y(2-x) = 3$, show that $3\frac{d^2y}{dx^2} - 2y\frac{dy}{dx} = 0.$

7. Given $y = (1 + 4x)e^{-2x}$, prove that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

8. Let $y = \sqrt{\cos x}$, show that $4y^3\frac{d^2y}{dx^2} + y^4 + 1 = 0.$

9. Given $(1 + x^2)y^2 = 1 - x^2$, show that $\left(\frac{dy}{dx}\right)^2 = \frac{1-y^4}{1-x^4}$.

10. Form a differential equation from $y = Ax^3 + \frac{B}{x^2} - 6x, \ x > 0$.

11. Form a differential equation from $y = Ax^3 + \frac{B}{x^2} - 6, \ x > 0$.

12. $y\sin^{-1}3x = \sqrt{1 - 9x^2}$, show that $(1 - 9x^2)\frac{dy}{dx} + 3y^2 + 9xy = 0$

13. Given that $y = x^n[A\cos(\ln x) + B\sin(\ln x)]$, where A and B are constants, show that

    $x^2\frac{d^2y}{dx^2} + (1 - 2n)x\frac{dy}{dx} + (1 + n^2)y = 0$

14. Given that $y = \sin^{-1}x$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

15. Let $y = 5e^{(\sqrt{3}-2)x} + 3e^{-(\sqrt{3}+2)x}$, show that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 0.$